flection can be accomplished and that its incorporation into the system will reduce the over-all weight and complexity of the system.

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Approximate Analytic Modeling of a Ballistic Aerobraking Planetary Capture

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Nomenclature

B = vehicle ballistic coefficient, $B = C_D A/2M$
R = flight-path radius of curvature
r = distance from planet center
s = distance along trajectory
V = velocity of vehicle
y = altitude, distance from planet surface
θ = true anomaly along trajectory
μ = gravitational constant
ρ = density of atmosphere

Introduction

SPACECRAFT approaching a planet may be captured by passing through the planet's atmosphere. Such maneuvers have been considered for Mars missions.[1,2] For the case of ballistic entry, simple analytic theories for the motion have been given.[3,4] This Note demonstrates the equivalence of these two theories, and gives a complete solution for the maximum deceleration due to drag in a form useful for preliminary mission planning.

The geometry of a ballistic aerobraking capture is given by Fig. 1. Maday[3] gives the vehicle equations of motion as

$$VdV/ds = -B\rho V^2 - (\mu/r^2)dr/ds, \quad V^2/R = (\mu/R)d\theta/ds$$

and these equations may be combined by noting that $d\theta/ds = (d\theta/dr)(dr/ds)$. It is now assumed that 1) the trajectory may be modeled by means of a drag-free (Keplerian) trajectory, and 2) the vehicle is in the atmosphere for small values of θ. The trajectory is then given by the expression

$$r = r_p\{1 + \tfrac{1}{2}[e/(1 + e)]\theta^2\} \tag{1}$$

where r_p is radius at perigee, and e is the eccentricity, given by $e = V_p{}^2 r_0/\mu - 1$, and r_0 is the planet radius. Density varies exponentially with altitude, $\rho = \rho_p \exp[-k(r - r_p)]$.

The equation of motion may be simplified for $|\theta| \ll 1$; the velocity change may then be found by quadratures, the integrations involving error integrals of large argument, which are

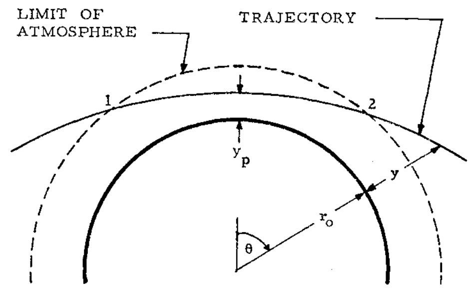

Fig. 1 Geometry of an aerobraking capture maneuver.

approximated by the complete error integral, with value $\pi^{1/2}/2$. In particular,[3]

$$(V_2/V_1) = \exp\{-B\rho_p[2\pi r_p(e + 1)/ke]^{1/2}\} \tag{2}$$

where k is the atmosphere scale height, and, if ρ_0 is the density at ground level, then $\rho_p = \rho_0 e^{-ky_p}$.

In contrast, the second theory[4] explicitly neglects gravity in the equation of motion:

$$dV/dt = -\rho BV^2 \tag{3}$$

The quantity m, the mass of gas "encountered" by the spacecraft, or lying within the tube of space swept out by its area A, is introduced as a parameter coupling the velocity reduction to the trajectory. The rate of mass encounter is $dm/dt = \rho AV$, and m is thus given by $m = (2M/C_D)\ln(V_1/V_2)$. Moreover, m is given as a function of the orbit shape, $r = r(\theta)$:

$$m = A \int_{r(\theta)} \rho(r)ds \tag{4}$$

As in the first theory, the orbit shape is given by Eq. (1), and the function $\rho = \rho(r)$ is taken as $\rho = \rho_0 e^{-ky}$. Since $|\theta| \ll 1$, $y \ll r_0$, $s \simeq r_0 d\theta$. The quadrature in Eq. (4) thus also involves error integrals of large argument. The problem considered in Ref. (4) is the inverse of that of Maday: given V_1 and V_2, to find y_p. The solution to this trajectory-design problem is given:

$$y_p = -\frac{1}{k}\ln\left[\frac{1}{B\rho_0}\left(\frac{k}{2\pi r_p}\,\frac{e}{1 + e}\right)^{1/2}\ln\frac{V_1}{V_2}\right] \tag{5}$$

where $r_p = r_0 + y_p$; $r_p \simeq r_0$. Comparing Eqs. (5) and (2), it is seen that they are equivalent.

Comparison of the Theories

The second method[4] explicitly neglects gravity; in this it resembles the Allen-Eggers[5] approximate theory for ballistic entry at high flight-path angles. Maday's theory explicitly takes account of gravity. However, Maday gives the influence of gravity as being zero; for the gravity term in his expression for V_2/V_1 involves the multiplier $(\theta_1 + \theta_2)$, and by the symmetry of Eq. (1), it is seen that this is zero, i.e. atmosphere entry and exit take place at the same value of $|\theta|$. Moreover, the second method allows the convenient study of trajectories whose shapes are given by expressions other than Eq. (1), which are to be used in Eq. (4). Orbits considered include a "double conic," with two branches each of which is given by an expression such as Eq. (1), but with different eccentricities for $\theta < 0$ and $\theta > 0$, and an expression similar to Eq. (1) but adding a term proportional to θ. It is found that differences in y_p from the value given by Eq. (5) are of the order of 1%.

Drag is proportional to ρV^2 and we require, for the drag to be maximum, that $d(\rho V^2)/dt = 0$, or $V^2(d\rho/dt) + 2\rho V(dV/dt) = 0$. Using Eq. (3),

$$d\rho/ds - 2B\rho^2 = 0 \tag{6}$$

We have $s \simeq r_0\theta$; let $\rho(s) = \rho_p \exp[-k(y - y_p)]$. Using

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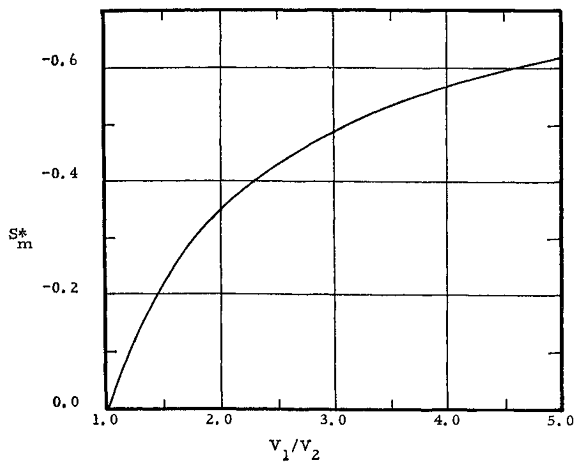

Fig. 2 S_m^*, dimensionless location of the point of maximum deceleration.

Eqs. (1) and (5),

$$\rho = \{(1/B)[ke/2\pi r_0(1+e)]^{1/2}\ln(V_1/V_2)\} \times \exp[-kes^2/2r_0(1+e)]$$

and Eq. (6) reduces to the transcendental algebraic equation

$$\ln[V_1/V_2][2r_0(1+e)/\pi ke]^{1/2}\exp[-kes^2/2r_0(1+e)] + s = 0 \tag{7}$$

Now define a dimensionless distance, $s^* = s[ke/2r_0(1+e)]^{1/2}$, and Eq. (7) becomes

$$[\pi^{-1/2}\ln(V_1/V_2)]e^{-s^{*2}} + s^* = 0$$

the solution of which is s_m^*, the point of maximum deceleration (Fig. 2). Note that there is only a single solution, $s_m^* < 0$.

Corresponding to s_m^* is the altitude y_m at maximum deceleration, $y_m = y_p + s_m^{*2}/k$. It is possible to find V_m by using either of the two theories outlined previously. It is found that

$$V_m/V_p = (V_1/V_2)^{(1/2)\mathrm{erf}(s_m^*)} \tag{8}$$

where erf(u) is the error function. The perigee velocity V_p is found by the symmetry of Eq. (1) to be simply, $V_p^2 = V_1V_2$. The deceleration at perigee is given by

$$(dV/dt)_p = V_1V_2[ke/2\pi r_0(1+e)]^{1/2}\ln(V_1/V_2)$$

and finally the maximum deceleration is given by

$$(dV/dt)_m/(dV/dt)_p = [V_m/V_p]^2e^{-s_m^{*2}} \tag{9}$$

Equations (8) and (9) are plotted in Fig. 3.

Figures 2 and 3 are thus useful for preliminary mission planning, since all perigee quantities are known as functions of parameters of the planet and of its atmosphere, and of V_1 and V_2 which are boundary conditions for the aerobraking maneuver. The location and value of $(dV/dt)_m$ is then purely a function of V_1/V_2, and is independent of all spacecraft design parameters as well as of the atmospheric density.

Although Maday[3] has outlined a solution to the determination of $(dV/dt)_m$, his work contains only an approximate solution to Eq. (7).

Conclusions

Two simple analytic theories for ballistic aerobraking capture are equivalent. Both ignore gravity effects, the first implicitly, the second explicitly; they must thus be regarded as different formulations of a common theory. The discussion of maximum deceleration goes beyond that of Maday by giving results in terms of dimensionless quantities; for a given

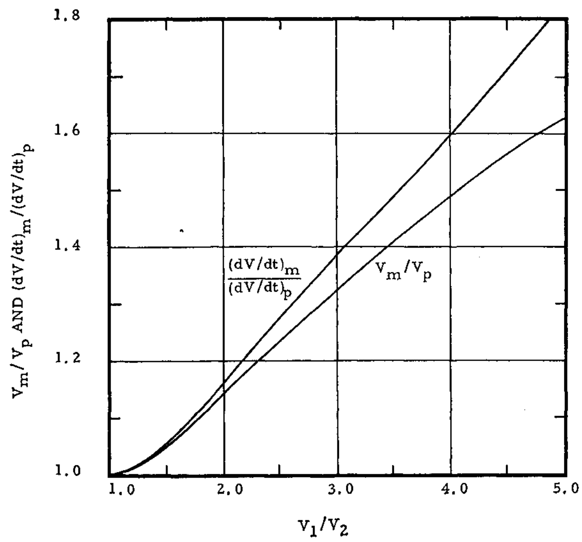

Fig. 3 Maximum deceleration and corresponding velocity, normalized with respect to deceleration and velocity at perigee.

mission, maximum deceleration is a function only of V_1/V_2, independent of ballistic coefficient. This result is analogous to a result given by Allen and Eggers in their atmosphere-entry theory.[5]

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Integrated Electronics for the Space Shuttle

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THIS Note summarizes the technical and cost goals and applicable technology improvements for the integrated electronics (avionics) of the space shuttle. The goal of the space shuttle program is to develop a low-cost reusable space transportation system for delivery of men and materials to

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